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**THE ANALYSIS OF NON-LINEAR DYNAMIC BEHAVIOR
(INCLUDING SNAP-THROUGH) OF POSTBUCKLED PLATES
BY SIMPLE ANALYTICAL SOLUTION**

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**THE ANALYSIS OF NONLINEAR DYNAMIC BEHAVIOR (INCLUDING SNAP-THROUGH)
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C. F. Ng

SUMMARY

Static postbuckling and nonlinear dynamic analyses of plates are usually accomplished by multimode analyses, although the methods are usually complicated and do not give straightforward understanding of the nonlinear behavior. Assuming single-mode transverse displacement, a simple formula is derived in this paper for the transverse load-displacement relationship of a plate under in-plane compression. The formula is used to derive a simple analytical expression for the static postbuckling displacement and nonlinear dynamic responses of postbuckled plates under sinusoidal or random excitation. Regions with softening and hardening spring behavior are identified. Also, the highly nonlinear motion of snap-through and its effects on the overall dynamic response can be easily interpreted using the single-mode formula. The theoretical results are compared with experimental results obtained using a buckled aluminum panel, using discrete frequency and broadband point excitation. Some important effects of the snap-through motion on the dynamic response of the postbuckled plates are found.

INTRODUCTION

If a plate is curved, initially or subsequently in service due to postbuckling stresses, the static and dynamic behavior in the transverse direction can be highly nonlinear, which may include hardening-spring, softening-spring or even an instability condition with snap-through motion. Theoretical and

experimental results of large amplitude vibration of postbuckled plates under sinusoidal excitation were obtained by Yamaki and Chiba,¹ however, snap-through motion was not studied. The characteristics of snap-through motion in a postbuckled beam under sinusoidal excitation was studied by Tseng and Dugundgi.² A theoretical study of the random response of an initially curved beam including snap-through motion was done by Seide.³ However, a thorough and straightforward understanding of the nonlinear behavior (particularly snap-through motion) of general curved plates is difficult to gather from the previous research results. The present study was conducted to fill this gap using a single-mode analysis method and experimental investigation with sinusoidal and random excitation forces on a postbuckled plate.

GENERAL FORMULAE FOR NONLINEAR BEHAVIOR OF PLATES

Equation for Equilibrium in the Transverse Direction

For a plate under uniaxial compression with uniform edge displacement, the relationship between modal displacement and modal force for the buckling mode is given by (derived in Appendix I):

$$\text{for static equilibrium:} \quad q^3 - Rq = p \quad (1a)$$

$$\text{for dynamic motion:} \quad \ddot{q}/\Omega^2 + 2\zeta\dot{q}/\Omega + (q^3 - Rq) = p \quad (1b)$$

where

q , nondimensional displacement parameter, $= Q/Q_p$

Q , modal displacement

Q_p , value of Q at $R = 1$

$R = \lambda - 1$

$\lambda = u/u_c$

u = in-plane edge shortening displacement

u_c = value of u at which buckling starts

Ω = linear natural circular frequency of the flat configuration

ζ = modal damping coefficient

p = nondimensional force parameter, $= \frac{P}{KQ_p}$

P = externally applied modal force

K = linear modal stiffness of the flat plate

Q_p , u_c , Ω , ζ , K depend on the assumed shape function of the mode and other plate parameters. The nondimensional parameters, q , R , P , can be evaluated after Q_p , u_c , K are found by experiments or theories. The equation (1a) involves only nondimensional parameters and is therefore independent of the plate parameters. Using equation (1), the nonlinear static and dynamic behaviors of a plate can be predicted and they are applicable to plates of any size, boundary conditions, material properties.

From the plot of P versus q for static condition (fig. 1) regions of hardening and softening spring behavior are found and there are also regions of negative stiffness, (e.g., between A and B). Notice that for $R = 1$, dynamic motion starting from C will pass through A and B and ends up at C'.

Static equilibrium positions are found by putting $p = 0$ in equation (1) and correspond to the points where the curve crosses the q -axis in figure 1. Note that for $R > 0$, there are three equilibrium values of q ; $R^{1/2}$, $-R^{1/2}$ and zero. The last value, zero, is an unstable position as the stiffness is negative. We can rewrite equation (1) as

$$q^3 - q_0^2 q = p \quad (2a)$$

$$\ddot{q}/\Omega^2 + 2\zeta\dot{q}/\Omega + (q^3 - q_0^2 q) = p \quad (2b)$$

where q_0 = equilibrium position = $R^{1/2}$.

From equation (2a), $dp/dq = 0$ when $q = \pm 1/\sqrt{3} q_0$, these are the end points of region of negative stiffness, e.g., A,B in figure 1

Undamped Free Vibration of Postbuckled Plate

From equation (2b), neglecting damping effect and external force, the free vibration is obtained as

$$\frac{1}{\Omega^2} \ddot{q} + (q^3 - q_0^2 q) = 0 \quad (3)$$

With the substitution $\ddot{q} = \dot{q} \frac{d\dot{q}}{dq}$, equation 3 can be integrated. For the initial conditions $\dot{q}(0) = 0$ and $q(0) = q_s$ (q_s is the initial position, $q_s > q_0$) at $t = 0$ the solution can be written

$$\dot{q}^2 = \frac{\Omega^2}{2} (q_s^2 - q^2) (q_s^2 + q^2 - 2 q_0^2) \quad (4)$$

With the expression $q_s^2 + q^2 - 2 q_0^2$ set equal to 1, the above formula is a standard linear vibration equation for initial value problem.

The other extreme point of vibration, after the system was released at q_s , q_e , is found by substituting $\dot{q} = 0$ in equation (4) to obtain

$$q_e = -q_s \quad \text{or} \quad q_e = (2 q_0^2 - q_s^2)^{1/2} \quad (5)$$

When $q_s < \sqrt{2} q_0$, $2 q_0^2 - q_s^2 > 0$, $2 q_0^2 - q_s^2 < q_0^2$ (since $q_s > q_0$), for the second solution, $0 < q_e < q_0$ and $q_0^2 - q_e^2 = q_s^2 - q_0^2$, thus the oscillation is around q_0 , from $q_s (> q_0)$ to $q_e (< q_0)$. The first solution $q_e = -q_s$ is on the negative side and thus not reached practically when $q_s < \sqrt{2} q_0$.

When $q_s > \sqrt{2} q_0$, $2q_0^2 - q_s^2 < 0$, the second solution gives an imaginary number, thus the only possible solution is $q_e = -q_s$, which means that the oscillation is from q_s to $-q_s$. Also the motion passes through both equilibrium positions of q_0 and $-q_0$ and $q = 0$ is the new mean position, instead of the original static value, q_0 .

When $q_s = \sqrt{2} q_0$, $2q_0^2 - q_s^2 = 0$, thus $q_e = 0$.

From (4), by substituting $q = \frac{dq}{dt}$, it can be written as.

$$dt = \frac{\sqrt{2} dq}{\Omega[(q_s^2 - q^2)(q_s^2 + q^2 - 2q_0^2)]^{1/2}} \quad (6)$$

By numerical integration of (6) for the motion between the extreme points, q_s , q_e , time histories for the free vibration of various amplitudes were determined for a plate with $q_0 = 1$ and are shown in figure 2. When $q_s < \sqrt{2}$, it can be seen that the period of vibration increases as amplitude increases and the motion is not symmetrical about the static value, 1. When $q = \sqrt{2} = 1.414$, the period is theoretically infinity as it takes infinite time to approach zero. However, when $q_s > \sqrt{2}$ the period will decrease with increase of amplitude. The displacement also passes through both equilibrium positions, $q_0 = 1$ and $q_0 = -1$, thus indicating snap-through motion. The change of mean position is q_0 (from q_0 to 0). Also the rms value is found to be approximately 1 when $q_s = 1.5$. Essentially, the postbuckled plate shows softening spring behavior initially and after snap-through motion accompanied by a change of equilibrium position it shows a hardening spring behavior.

The free vibration response characteristics reported in reference 2 also shows that resonance frequency decreases to zero when snap-through motion is initiated and the frequency increases when the magnitude of snap-through motion gets larger.

Random Vibration

The method of equivalent linearization can be used to solve the nonlinear forced vibration equation with damping (from eq. (2b));

$$\frac{1}{\Omega^2} \ddot{q} + 2 \frac{\zeta}{\Omega} \dot{q} + (q^3 - q_0^2 q) = p \quad (7)$$

The mean square displacement of a buckled plate, $\langle q^2 \rangle$ due to white noise excitation with spectral density S_{pp} can be obtained for small and large magnitudes, as described in Appendix II.

For small excitations $\alpha < 0.45 q_0^4$ ($\alpha = \pi \Omega S_{pp} / 4\zeta$, α is a nondimensional force parameter), there is no snap-through motion and

$$\langle q^2 \rangle = [-q_0^2 + (q_0^4 + 3\alpha)^{1/2}] / 3, \text{ and } \bar{q} \approx q_0. \quad (8)$$

For $\alpha \geq 2 q_0^4$ there is persistent snap-through motion in almost every cycle of oscillation and

$$\langle q^2 \rangle = [q_0^2 + (q_0^4 + 12\alpha)^{1/2}] / 6, \text{ and } \bar{q} = 0. \quad (9)$$

For $0.45 q_0^4 \leq \alpha < 2 q_0^4$, snap-through motion is intermittent, the mean position as well as the mean square values are very unsteady. However, the mean square value can be taken approximately from interpolation between the two end points--the point of no snap-through and persistent snap-through motion.

From equations (8) and (9) the variation of $\langle q^2 \rangle$ with the excitation parameter α for different values of compression parameter λ is shown in figure 3. The rate of increase of response with excitation is highest when intermittent snap-through motion starts. When persistent snap-through motion

is attained, the response increases much more slowly with increases in excitation, showing hardening spring behavior.

From figure 3 and equations (8) and (9), the variation of response with compression parameter for various levels of excitation are plotted as shown in figure 4. For a given excitation level, the response increases with increases in compression as it approaches the buckling point. After initial post-buckling, persistent snap-through occurs and the response continues to increase until a certain point for which only intermittent snap-through motion can be induced. After that point, the response decreases with further increases in compression. The point of maximum displacement corresponds to the point for which the excitation is just sufficient for persistent snap-through motion. Also, the point of maximum response occurs at increasingly greater plate curvatures, or larger values of u/u_c , as the excitation level increases. These trends have also been predicted in a qualitative description by Jacobson.⁵

EXPERIMENTAL RESULTS

The Test Set-Up

Dynamic tests on several 0.032-inch thick aluminum plates were carried out using point excitation at the center of the plate by an electromagnetic shaker. Displacement response was measured by strain gauges. As shown in figure 5, excitation force was applied in both directions but without applying any bending constraint on the plate using rounded point screws connected together to the shaker by a rectangular frame around the specimen. This direct attachment method of excitation was used instead of the base excitation of the supporting frame because it can ensure large excitation force in the low-frequency range (0-20 Hz).

Results for Discrete Frequency Excitation

In a preliminary test, it was found that snap-through motion was most readily excited by an excitation frequency of 5 Hz. The variation of the rms value of strain response and reduction of the mean value of strain due to snap-through with excitation level are plotted in figure 6. There is a region of unsteady response as snap-through motion is initiated. When more power is put into the shaker, the excitation force decreases with increase of response. This is accompanied by a comparable reduction in the mean value of the oscillation approximately equal to the original static value, as predicted in figure 2.

Random Responses

Broadband (0-100 Hz) excitation was used to excite random response (fig. 7). The low-frequency response (0-20 Hz) dominates the strain response when snap-through motion is initiated at $u/u_c = 4.0$ (fig. 7) and the fundamental modal response is not evident. The dominance of low-frequency response for large amplitudes of post-buckled plates was also reported in references 5 and 6.

Figure 8 shows the transfer functions (strain/force) of a flat plate with no buckling load, $u/u_c = 0$, for various excitation levels. The resonance peak broadens and the overall level decreases as excitation level is increased. This is a typical hardening spring behavior.

Figure 9 shows transfer functions of a buckled plate. The resonance peak also broadens and decreases as did the flat plate, but the response at low frequency increases as excitation level is increased. Increases in the low-frequency response are due to the onset of snap-through motion for which the natural frequency is very low (fig. 2).

The variation of mean square strain parameter (square of the ratio of dynamic strain to static strain at $R = 1$) with compression parameter is shown in figure 10. The general trend agrees well with predicted results from the single-mode formula (fig. 4) and the points of maximum responses are near the curve for static values. However, there is a large discrepancy between the experimental results and theoretical prediction for an excitation level of $\alpha = 6$, which indicates that the single-mode representation used in the analysis over-predicts the stiffness of the buckled plate. More modes may be required to represent the deformation pattern and give a lower overall stiffness value.

CONCLUSIONS

A simple formula was derived for the transverse load-displacement relationship of a plate under in-plane compression and compared with results from experiment using electromagnetic excitation method on an aluminum plate with postbuckling deflection. The comparison shows that the simple formula predicts the general trend of the highly nonlinear behavior of snap-through motion under dynamic excitation. The general characteristics of snap-through motion are:

- (1) The mean position of the oscillation is zero;
- (2) The r.m.s displacement value is approximately equal to the static equilibrium value of the plate when snap-through motion is just initiated;
- (3) Snap-through motion is most readily excited by low-frequency excitation; and
- (4) For a given random excitation level, the maximum response is found in the postbuckling configuration when the excitation is just sufficient for persistent snap-through motion.

The identification of the nonlinear characteristics found in post-buckled plates should be very helpful in studying the corresponding characteristics in other curved plates such as cylindrical panels.

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APPENDIX I

Derivation of Single-Mode Formula

For Any Plate Under Compression

The starting point is the Von Karman equations of large deflection of plates. The tranverse equilibrium equation and the in-plane comptability equation can be expressed, respectively, as

$$D\nabla^4 w = h + t \left(\sigma_x \frac{\partial^2 w}{\partial x^2} + \sigma_y \frac{\partial^2 w}{\partial y^2} + 2\tau_{xy} \frac{\partial^2 w}{\partial x \partial y} \right) \quad (A-1)$$

and

$$\frac{1}{E} \nabla^4 F = \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \quad (A-2)$$

where

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2}, \quad \tau_{xy} = - \frac{\partial^2 F}{\partial x \partial y}$$

F = Airy stress function

$\sigma_x, \sigma_y, \tau_{xy}$ = in-plane stresses

w = transverse displacement

D = plate flexural stiffness = $Et^3/12(1-\nu^2)$

E = modulus of elasticity

t = plate thickness

ν = Poisson's ratio

h = transverse load per unit area, i.e. the pressure.

The steps are the same as in the Rayleigh-Ritz method described in reference 4 and summarized as follows:

1. Transverse Displacement:

$$w(x,y) = tQ\phi(x)\psi(y) \quad (A-3)$$

where Q is the modal displacement coefficient

$\phi(x) \psi(y)$ is the buckling mode shape function.

The corresponding modal force coefficient, P , is

$$t \int_0^b \int_0^a h(x,y) \phi(x) \psi(y) dx dy .$$

2. Stress Function: substituting w from (A-3) into (A-2),

$$F = Et^2 Q^2 \sum_i \sum_j f_{ij} \phi_i(x) \psi_j(y) - \frac{1}{2} P_x^2 - \frac{1}{2} P_y^2 \quad (A-4)$$

where

$\phi_i(x)$, $\psi_j(y)$ are higher order functions related to $\phi(x)$, $\psi(y)$,

respectively, f_{ij} depends on i,j (details in ref. 4); and

P_x , P_y are mean compressive stresses in the x and y direction.

3. Mean Compressive Stresses: for the edge displacement

$$u = \int_0^a \left[\left(\frac{\sigma}{E} x - \nu \frac{\sigma}{E} y \right) - \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right] dx \quad (A-5)$$

From (A-4) and (A-5),

$$P_x = P_u - \frac{E}{(1-\nu^2)} C_{xy} Q^2 \quad (A-6)$$

where

$$P_u = \frac{E}{(1-\nu^2)} \left(\frac{u}{a} \right) ,$$

P_u is the mean compressive stress due to edge shortening if the plate is flat and C_{xy} is a constant related to $\phi(x)$, $\psi(y)$.

Similarly, with edge displacement in y direction being zero,

$$P_y = \nu P_x - E C_y Q^2 \quad (A-7)$$

C_y is a constant related to $\psi(y)$.

4. In-plane Strain Energy:

$$V_e = \int_0^b \int_0^a \frac{t}{2E} [\sigma_x^2 + \sigma_y^2 - 2\nu \sigma_x \sigma_y + 2(1 + \nu) \tau_{xy}^2] dx dy \quad (A-8)$$

From (A-4), (A-6) and (A-8),

$$V_e = \frac{tab}{2E} (P_x^2 + P_y^2 - 2\nu P_x P_y) + Et a b e Q^4 \quad (A-9)$$

where e is a constant related to f_{ij} in (A-4).

5. Bending Strain Energy:

$$V_b = \frac{Dt}{2} \int_0^b \int_0^a \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 dx dy \quad (A-10)$$

From (A-3) and (A-10),

$$V_b = DQ^2 d \quad (A-11)$$

where d is a constant related to $\phi(x)$, $\psi(y)$.

6. Linear Static Equilibrium Equation:

$$\frac{\partial V_b}{\partial Q} = P$$

This equation applies for flat plate with small value of Q , for which V_e can be neglected.

Substituting (A-9) into the above equation,

$$2DdQ = P$$

defining $K = 2Dd$, the linear modal equilibrium equation is obtained.

$$KQ = P \quad (A-12)$$

K is thus the linear modal stiffness.

7. Nonlinear Static Equilibrium Equation:

$$\frac{\partial(Ve + Vb)}{\partial Q} = P \quad (A-13)$$

Substituting from steps 4 and 5 and rearranging.

$$Q \left[\left(2e + \left(\frac{C_{xy}}{1 - \nu^2} \right)^2 + C_y^2 \right) \frac{Etab}{Dd} Q^2 - \frac{P_u tab C_{xy}}{Dd} + 1 \right] 2Dd = P \quad (A-14)$$

Substituting $\frac{Dd}{tab C_{xy}} = P_c$

$2Dd = K$, K is the linear modal stiffness from (A-12).

$$\left(2e + \left(\frac{C_{xy}}{1 - \nu^2} \right)^2 + C_y^2 \right) \frac{Etab}{Dd} = \frac{1}{Q_p^2}$$

the equation (A-14) becomes

$$Q \left[\frac{Q^2}{Q_p^2} - \left(\frac{P_u}{P_c} - 1 \right) \right] K = P \quad (A-15)$$

This is the nonlinear modal equilibrium equation. Q^2/Q_p^2 is the effect of large displacement, P_u/P_c is the effect of compression.

Defining $\frac{E}{(1 - \nu^2)} \frac{u_c}{a} = P_c$, and $\frac{P_u}{P_c} = \frac{u}{u_c}$, substituting $q = Q/Q_p$, $p = P/KQ_p$,

The general nondimensional equation is obtained by dividing (A-15) by KQ_p .

$$q^3 - (u/u_c - 1) q = p \quad (A-16)$$

Putting $p=0$ for loading with in-plane compression only,

For $\frac{P_u}{P_c} < 1$ ($\frac{u}{u_c} < 1$), $q = Q = 0$ (A-17)

For $\frac{P_u}{P_c} > 1 (u_c > 1), \frac{Q^2}{Q_p^2} = (\frac{P_u}{P_c} - 1)$

or $q^2 = (\frac{u}{u_c} - 1)$ (A-18)

From the above it can be seen that P_c and u_c are the critical mean compressive stresses and edge shortening respectively, thus when

$$\frac{u}{u_c} = 2 \quad Q = Q_p \quad \text{or} \quad q = 1 \quad (\text{A-19})$$

Therefore, Q_p is the value of Q when $u/u_c = 2$

8. Kinetic Energy

$$V_T = \frac{1}{2} \rho t \int_0^b \int_0^a \dot{w}^2 dx dy \quad (\text{A-20})$$

where ρ is the density.

Substituting (A-3) to (A-20)

$$V_T = \frac{1}{2} M \dot{Q}^2 \quad (\text{A-21})$$

where $M = \rho t^3 \int_0^b \int_0^a \phi^2(x) \psi^2(y) dx dy$ M is the modal mass.

9. Lagrangian Equation for Linear Vibration

$$\frac{\partial V_b}{\partial Q} + \frac{\partial}{\partial t} \frac{\partial V_T}{\partial \dot{Q}} = P \quad (\text{A-22})$$

Using results from (A-12), (A-20), the equation of motion is obtained,

$$KQ + M\ddot{Q} = P \quad (\text{A-23})$$

The natural linear circular frequency of the flat configuration is thus given by

$$\Omega^2 = K/M \quad (\text{A-24})$$

The modal mass M can be written in terms of Ω

$$M = \frac{K}{\Omega^2} \quad (A-25)$$

10. Lagrangian Equation for Nonlinear Vibration

$$\frac{\partial(V_e + V_b)}{\partial Q} + \frac{\partial}{\partial t} \frac{\partial V_T}{\partial Q} = P \quad (A-26)$$

using results from (A-15) and (A-23) the modal equation of motion is obtained,

$$\left[\frac{Q^2}{Q_p^2} - \frac{P_u}{P_c} - 1 \right] KQ + M\ddot{Q} = P \quad (A-27)$$

Dividing both sides by KQ_p and using results from (A-16) and (A-25), the nondimensional equation of motion is obtained,

$$q^3 - Rq + \frac{1}{\Omega^2} \ddot{q} = p \quad (A-28)$$

The damping effect can be similarly included by dividing the modal damping force $2M\zeta\dot{Q}$ (ζ is the modal damping coefficient) by KQ_p , which gives $\frac{2\zeta}{\Omega} \dot{q}$. Thus the nondimensional equation of motion can be written as

$$\frac{\ddot{q}}{\Omega^2} + \frac{2\zeta\dot{q}}{\Omega} + q^3 - Rq = p \quad (A-29)$$

APPENDIX II

Derivation of Nonlinear Acoustic Response Using Equivalent Linearization

1. Flat Plate

The linear equation of motion for the flat plate is

$$\frac{1}{\Omega^2} \ddot{q} + 2 \frac{\zeta}{\Omega} \dot{q} + q = p \quad (\text{B-1})$$

The mean square value of q , $\langle q^2 \rangle$, due to white noise excitation with

spectral density S_{pp} is given by $\langle q^2 \rangle = \frac{\pi \Omega S_{pp}}{4\zeta} = \alpha$, where α is a
nondimensional excitation parameter.

2. Postbuckled Plate without Snap-Through Motion

The nonlinear equation of motion for the postbuckled plate is

$$\frac{1}{\Omega^2} \ddot{q} + 2 \frac{\zeta}{\Omega} \dot{q} + (q^3 - q_0^2 q) = p \quad (\text{B-2})$$

Substituting $q = q_0 + \Delta q$ into equation (B-2), (Δq is the dynamic displacement around the static value q_0), the equation become

$$\frac{1}{\Omega^2} \ddot{\Delta q} + 2 \frac{\zeta}{\Omega} \dot{\Delta q} + (\Delta q^3 + 3\Delta q^2 q_0 + 2\Delta q q_0^2) = p \quad (\text{B-3})$$

The equivalent linear equation is

$$\frac{1}{\Omega^2} \ddot{\Delta q} + 2 \frac{\zeta}{\Omega} \dot{\Delta q} + k \Delta q = p \quad (\text{B-4})$$

where

$$\begin{aligned} k &= \frac{\partial(\Delta q^3 + 3\Delta q^2 q_0 + 2\Delta q q_0^2)}{\partial \Delta q} \\ &= 3 \langle \Delta q^2 \rangle + 2q_0^2 \end{aligned}$$

Thus from eq. (B-4)

$$\langle \Delta q^2 \rangle = \frac{\pi \Omega S_{pp}}{4 \zeta k} = \frac{\alpha}{3 \langle \Delta q^2 \rangle + 2 q_0^2} \quad (B-5)$$

Solving eq. (B-5) for $\langle \Delta q^2 \rangle$

$$\langle \Delta q^2 \rangle = \left[-q_0^2 + \left(q_0^4 + 3\alpha \right)^{\frac{1}{2}} \right] / 3 \quad (B-6)$$

3. Postbuckled Plate with Snap-Through Motion

The mean position of q , is zero, thus equation (B-2) is used directly and the equivalent linear equation is

$$\frac{1}{\Omega^2} \ddot{q} + 2 \frac{\zeta}{\Omega} \dot{q} + kq = p \quad (B-7)$$

where $k = \frac{\partial(q^3 - q_0^3 q)}{\partial q} = 3 \langle q^2 \rangle - q_0^2$

From eq. (B-7)

$$\langle q^2 \rangle = \frac{\pi \Omega S_{pp}}{4 \zeta k} = \frac{\alpha}{3 \langle q^2 \rangle - q_0^2} \quad (B-8)$$

Solving eq. (B-8) for $\langle q^2 \rangle$

$$\langle q^2 \rangle = \left[q_0^2 + \left(q_0^4 + 12 \alpha \right)^{1/2} \right] / 6 \quad (B-9)$$

4. The Excitation Level for Snap-Through Motion

For (B-9) to be valid, $\langle q^2 \rangle \geq q_0^2$ so that most of the oscillation consist of complete snap-through motion (the r.m.s. of which is approximately q_0 , as shown in fig. 2). By substituting $\langle q^2 \rangle$ from (B-9) to $\langle q^2 \rangle \geq q_0^2$ $\alpha > 2 q_0^4$ is obtained as the condition for persistent snap-through motion.

If $\langle q^2 \rangle < 1/3 q_0^2$, the oscillation rarely gets into the region of negative stiffness (between A and B in fig. 1), there is no snap-through motion. By substituting $\langle q^2 \rangle$ from (B-6) to $\langle q^2 \rangle < 1/3 q_0^2$, we obtain $\alpha < 0.45 q_0^4$ is obtained as the condition for no snap-through motion.

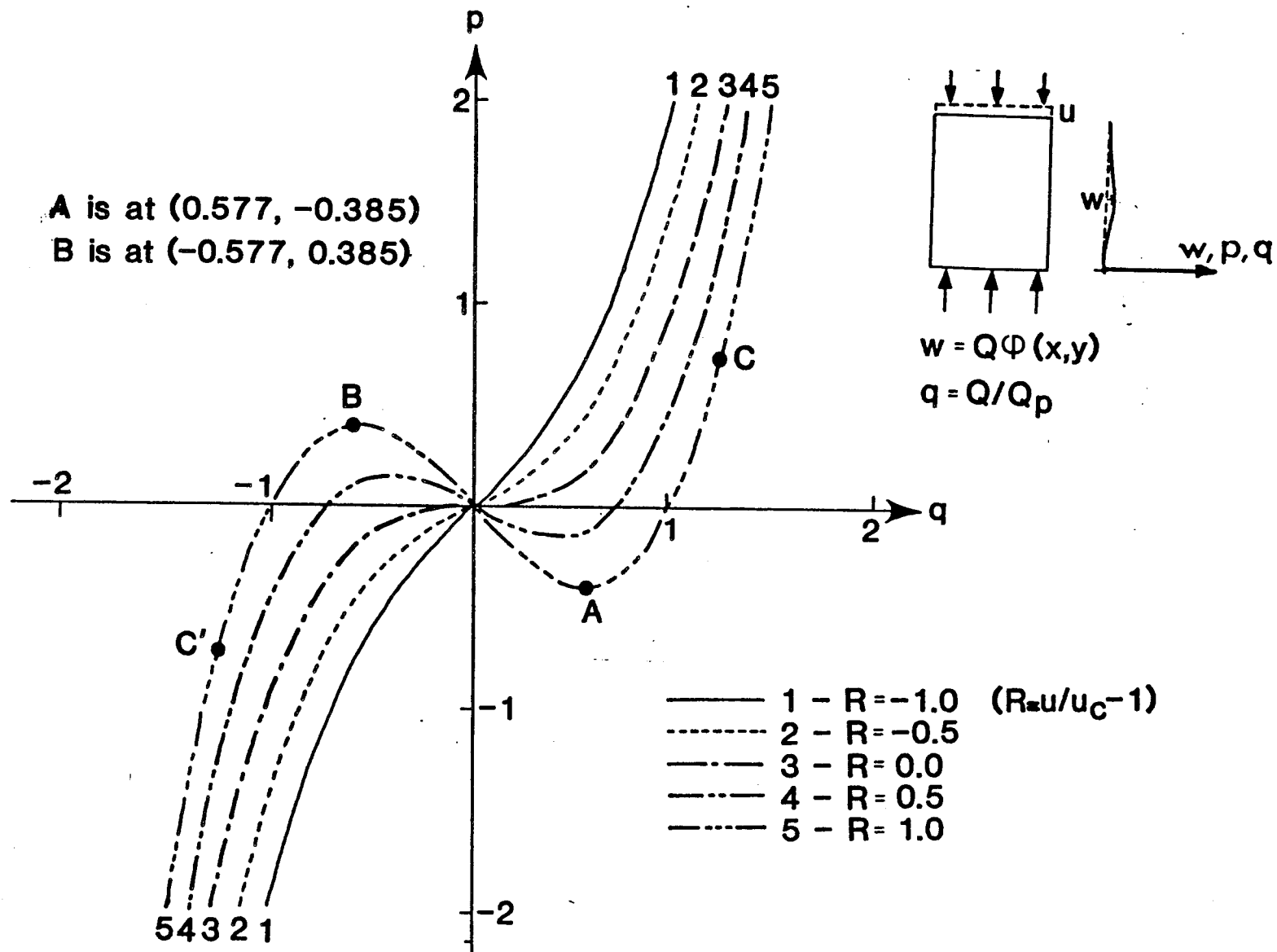


Figure 1. Plots for transverse force equilibrium equation of a plate under in-plane compression.

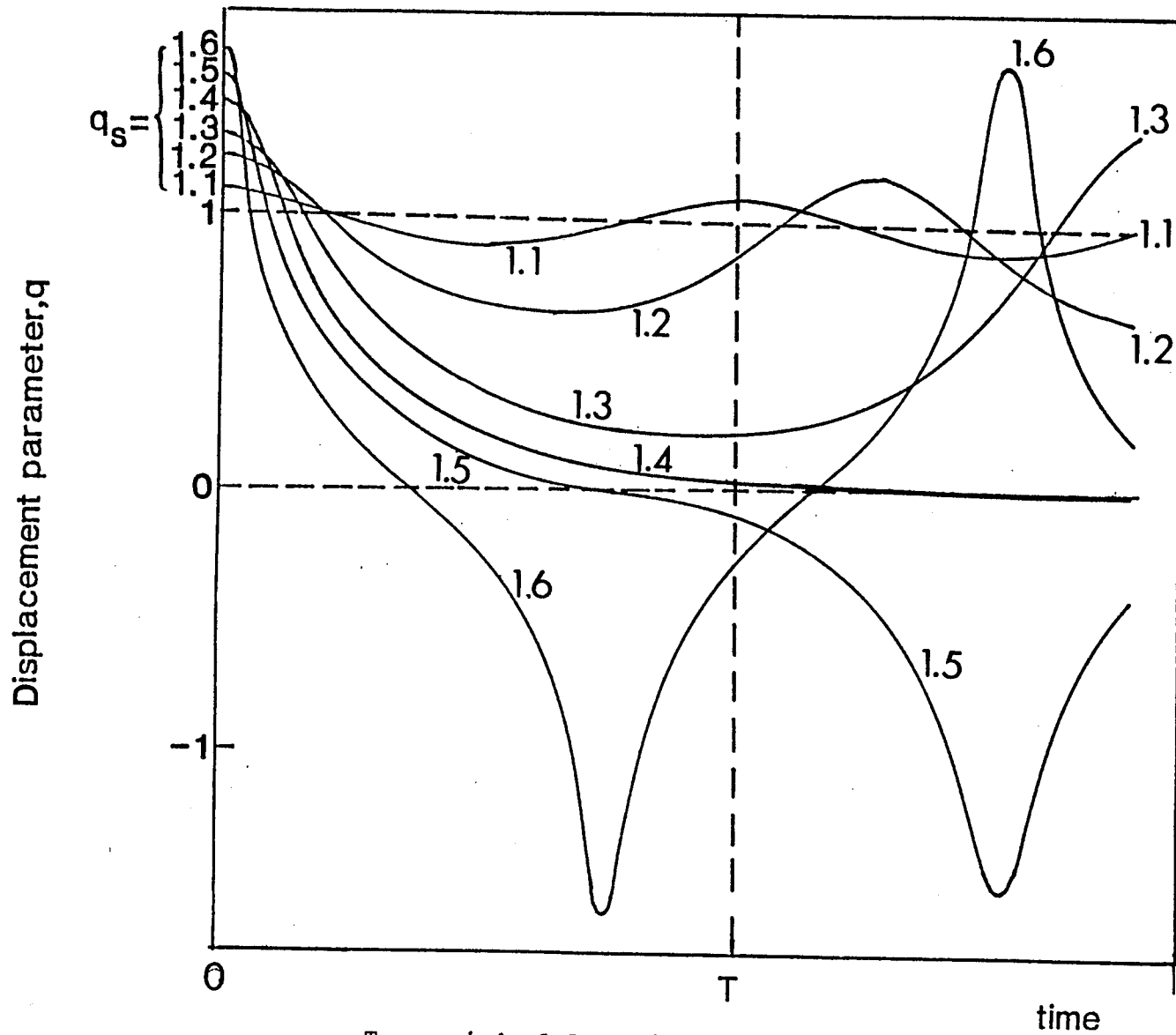


Figure 2. The free vibration of a postbuckled plate for different magnitudes of initial displacement, q_s . $R = 1$ ($u/u_c = 2$), $q_0 = 1$.

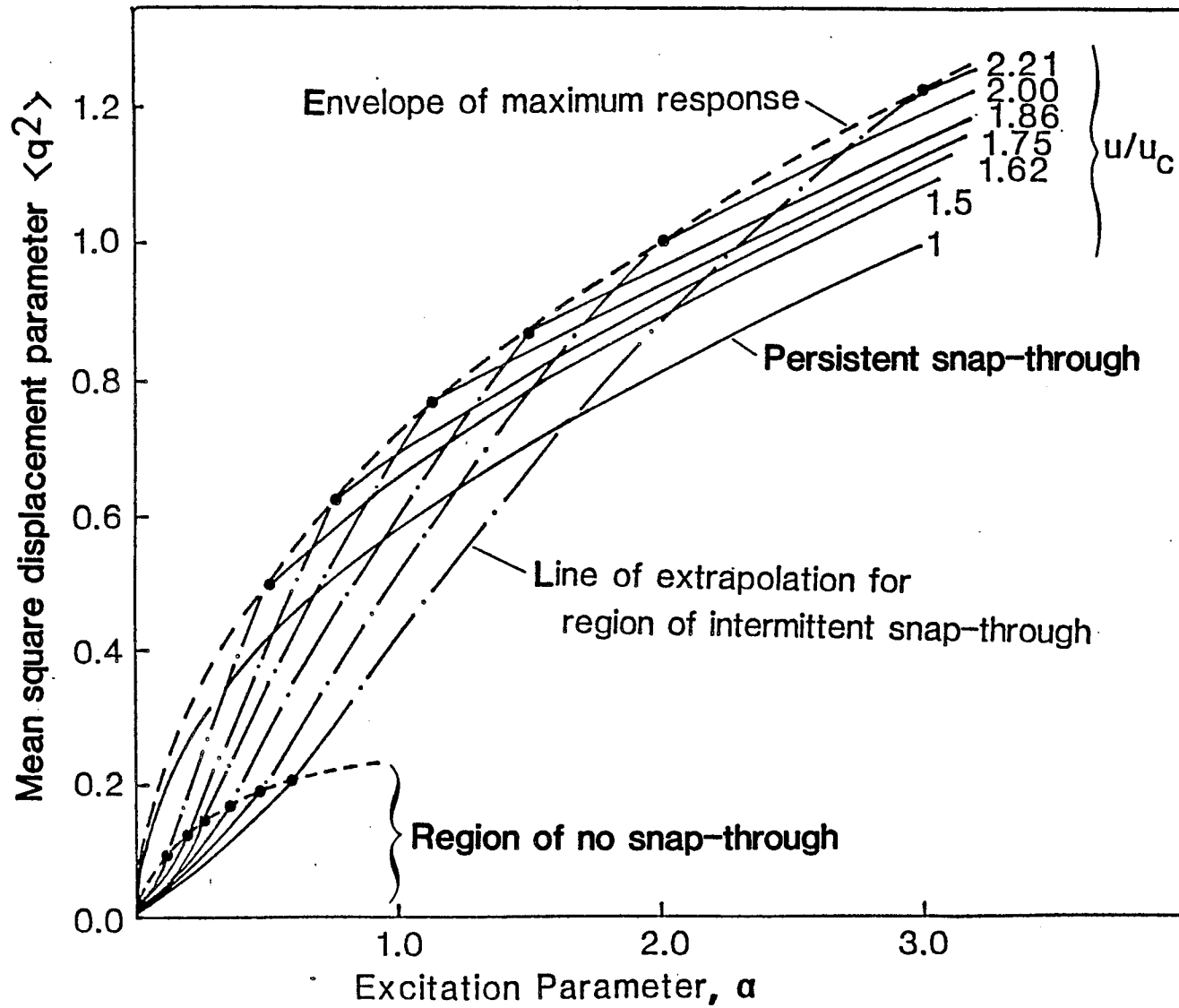


Figure 3. The variation of displacement response with excitation level in a postbuckled plate.

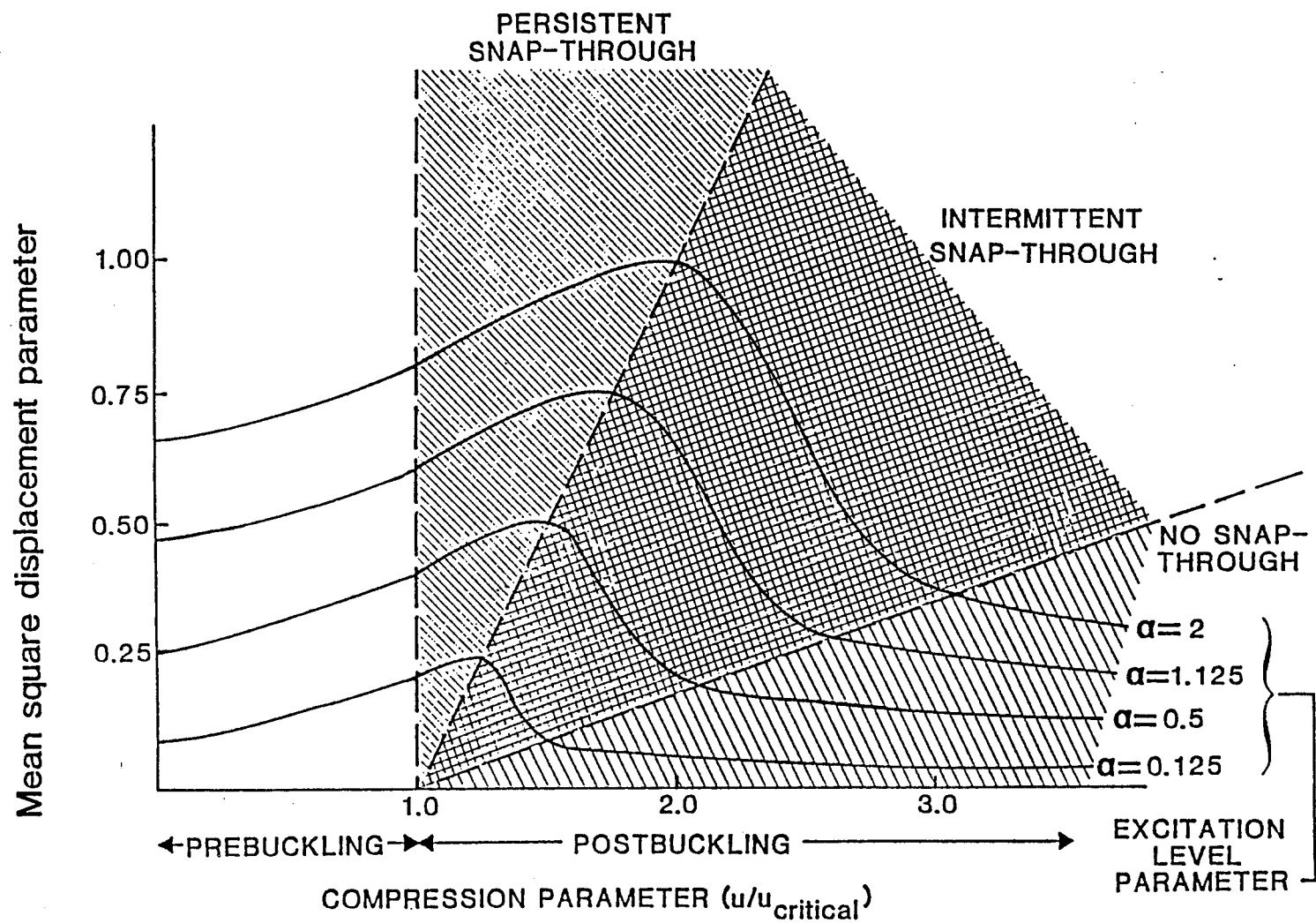


Figure 4. The variation of displacement response of postbuckled plates under white noise excitation.

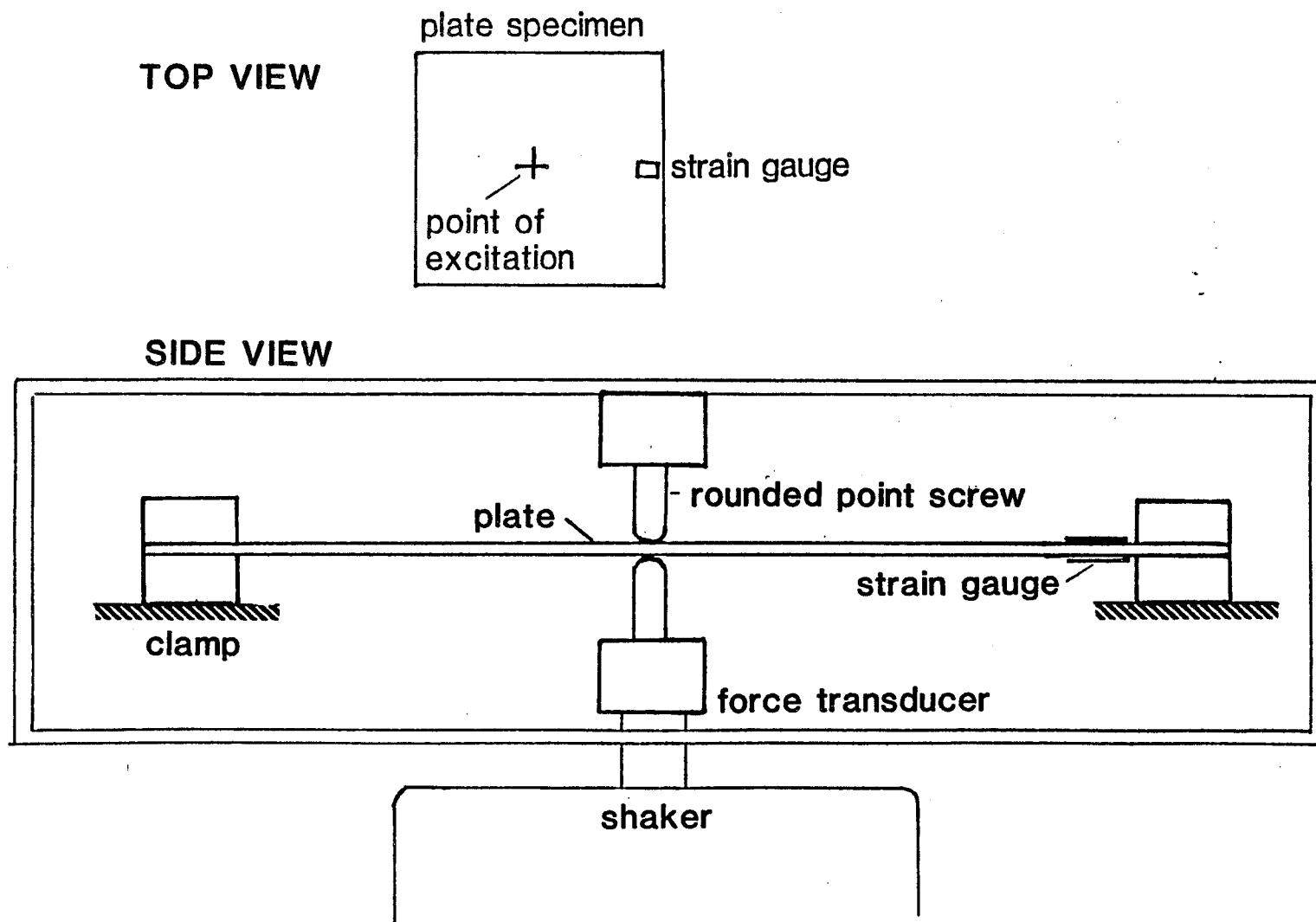
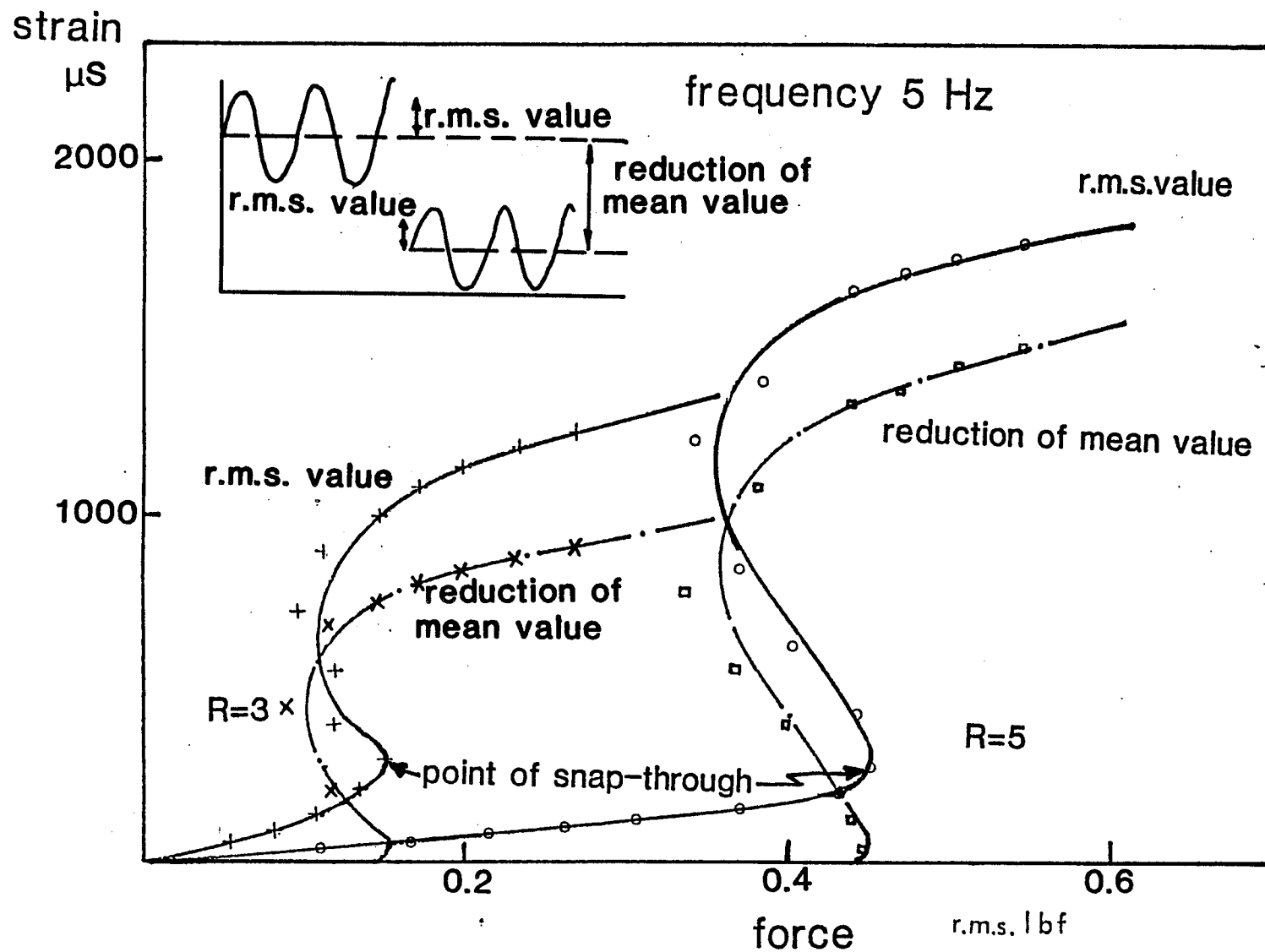


Figure 5. The schematic diagram for the dynamic test of a plate.



for R=3, static strain = 1050 μ in/in.

for R=3, static strain = 1550 μ in/in.

Figure 6. The variation of strain response with excitation level in a plate.

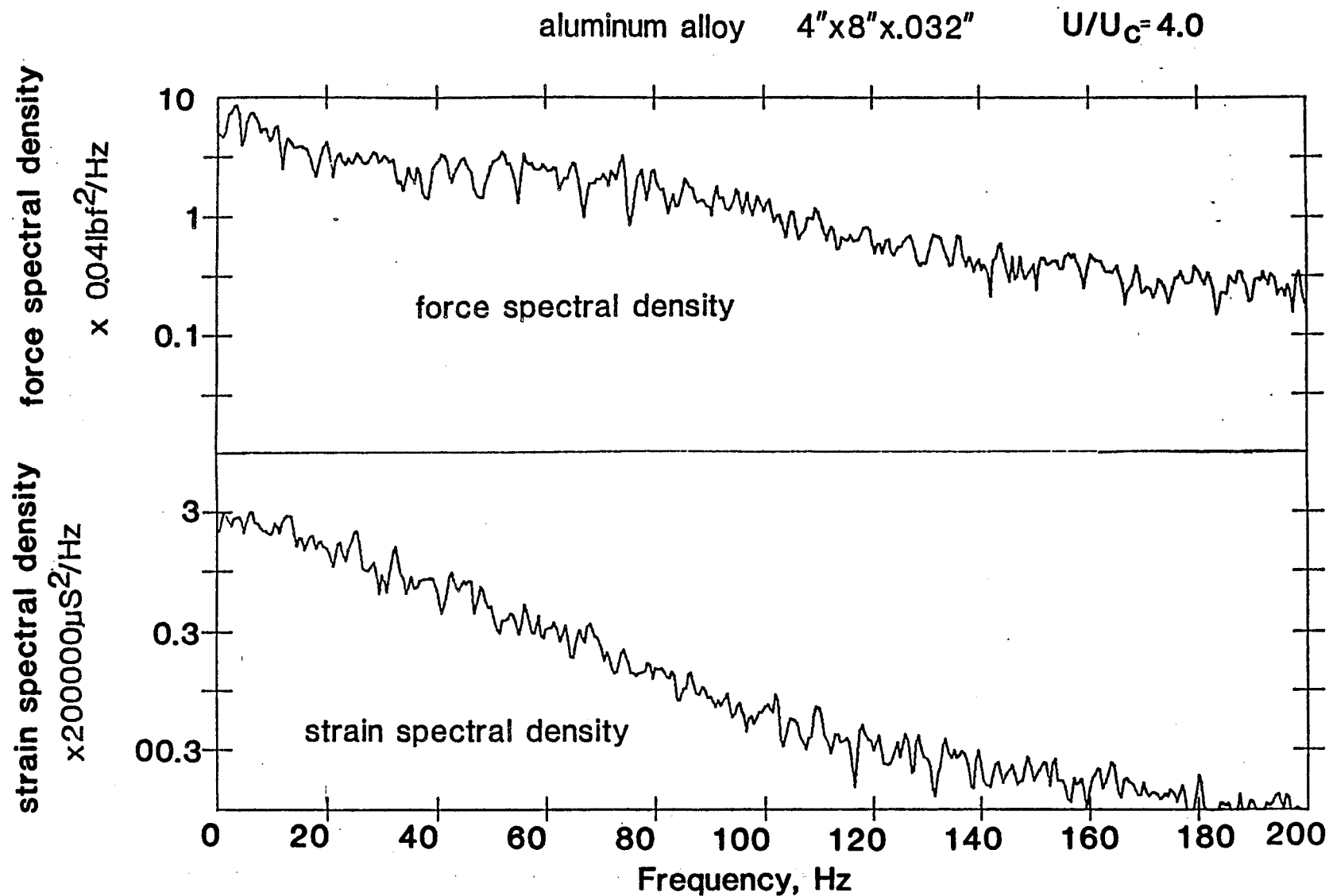


Figure 7. The strain response to random excitation in a postbuckled plate with persistent snap-through.

Cross spectral density of strain and force/
spectral density of force

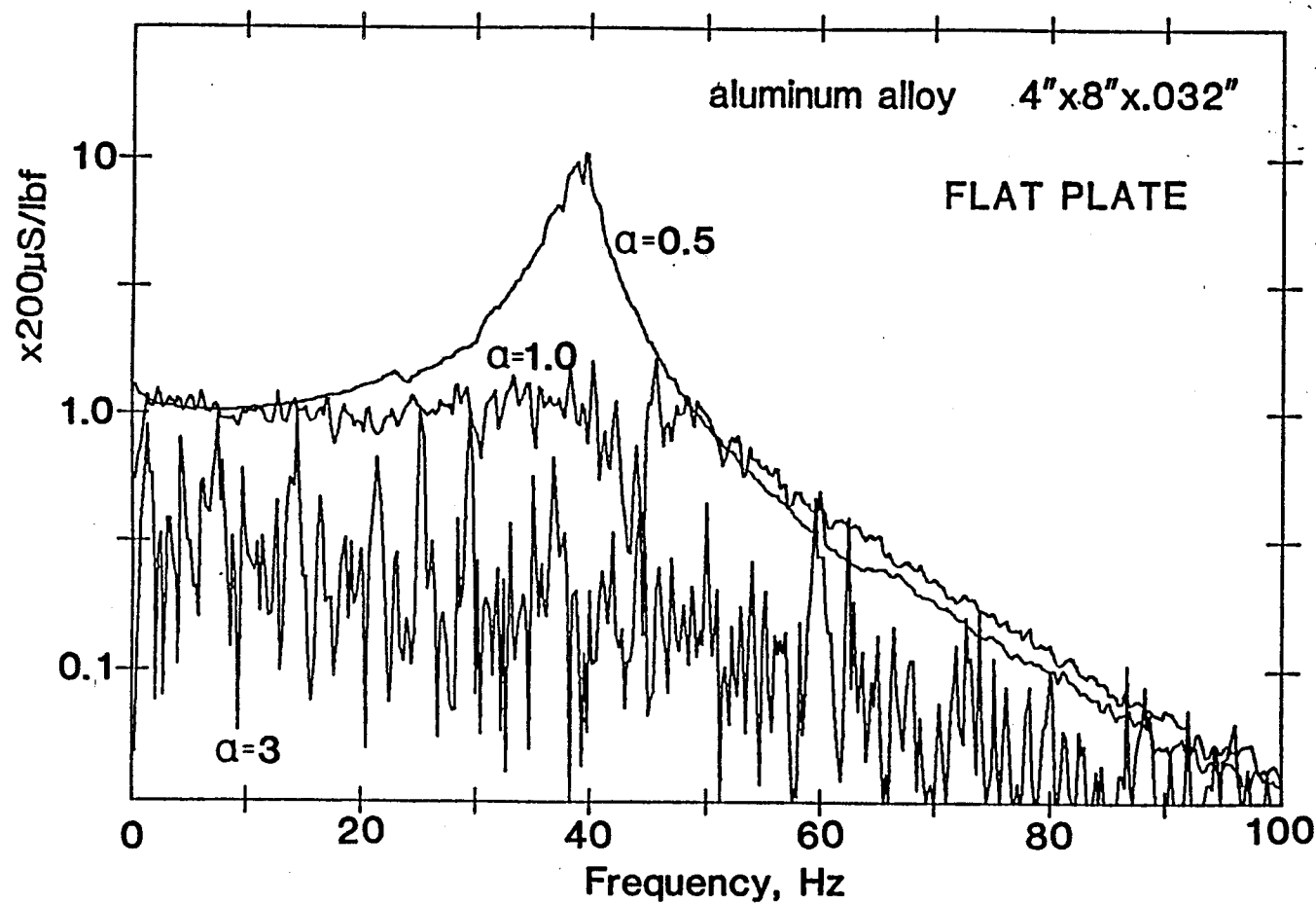


Figure 8. The transfer function of the strain response of an isotropic plate under random excitation (no compression load $u/u_c = 0.0$).

Cross spectral density of strain and force/ spectral density of force

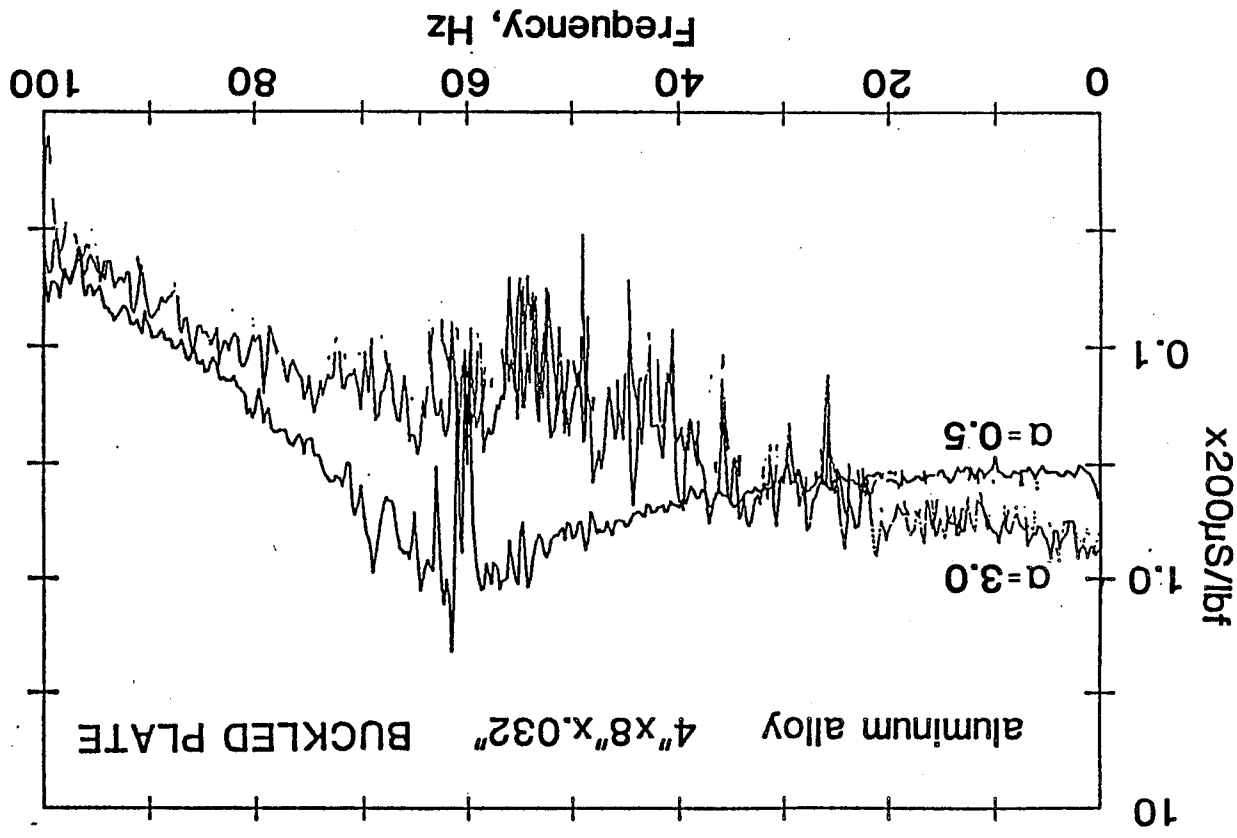


Figure 9. The transfer function of the strain response of an isotropic plate under random excitation ($u/u_c = 5.0$).

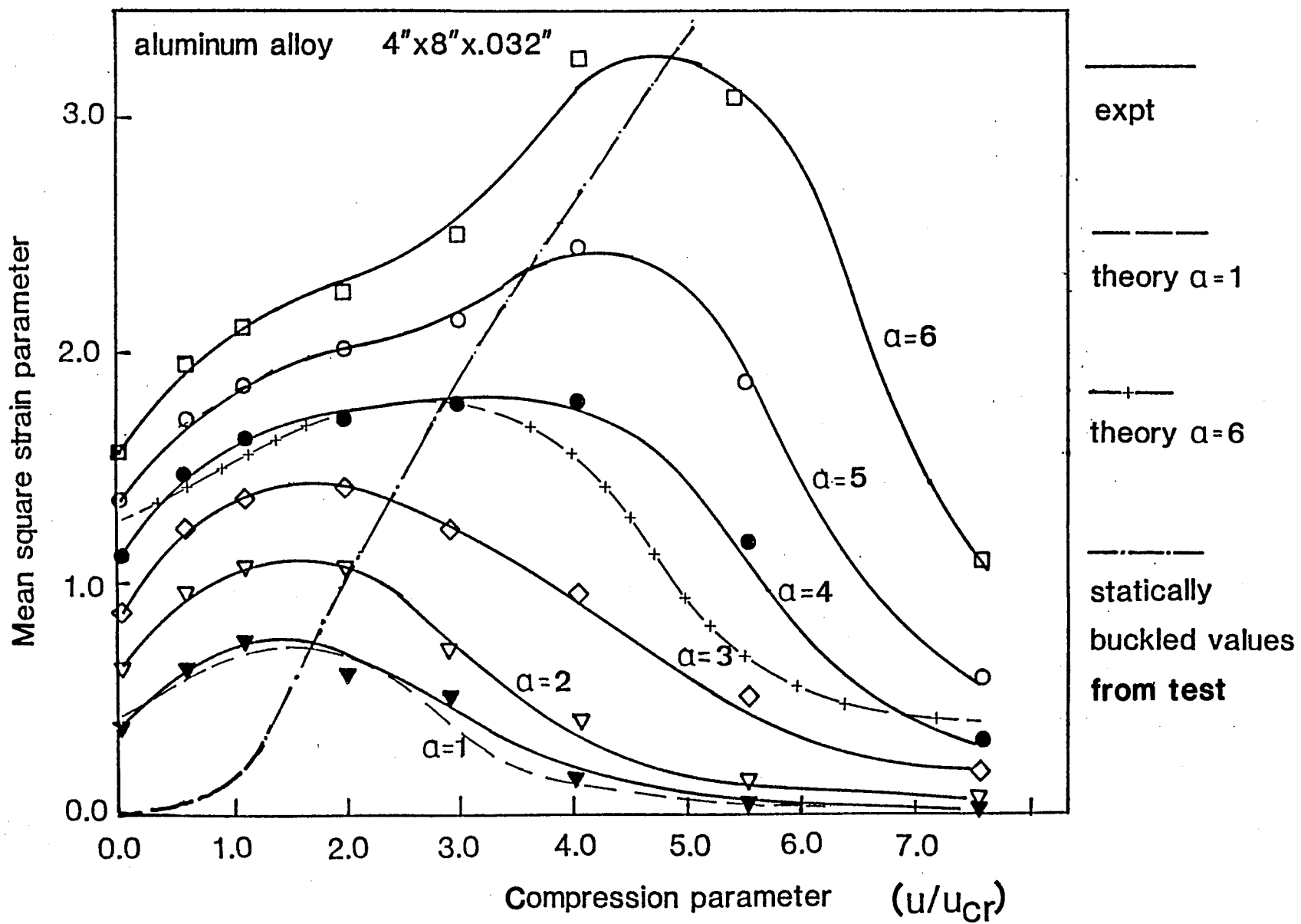


Figure 10. Variation of random strain response with increasing compression in an isotropic plate.

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16. Abstract Static postbuckling and non-linear dynamic analysis of plates had been done usually by multiple analyses but these methods are usually complicated and do not give straight forward understanding of the non-linear behavior. Assuming single-mode transverse displacement, a simple formula is derived for the transverse load-displacement relationship of a plate under in-plane compression and it is applicable to any plate of any boundary condition and aspect ratio. The formula is used to derive simple analytical expression for the static postbuckling displacement, and non-linear dynamic responses of postbuckled plates under sinusoidal or random excitation. Regions with softening and hardening spring behavior are identified. Also, the highly non-linear motion of snap-through and its effects on the overall dynamic response can be easily interpreted using the single-mode formula. The theoretical results are compared with experimental results obtained in postbuckled aluminum specimens using point excitation of discrete frequency and broadband. Some important effects of the snap-through motion on the dynamic response of the postbuckled plates are found.					
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